FMS Sea Ice Simulator

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Abstract

This document describes a sea ice simulator (SIS) for use in coupled models under the FMS framework. FMS is the Flexible Modelling System developed at NOAA's Geophysical Fluid Dynamics Laboratory. The system is designed to be modular so that SIS may be easily replaced in an FMS coupled model by another sea ice model following the FMS coupling interface standard. SIS calculates the concentration, thickness, temperature, brine content, and snow cover of an arbitrary number of sea ice thickness categories (including open water) as well as the motion of the complete pack. Additionally, the model is responsible for calculating ice/ocean fluxes and communicating fluxes between the ocean and atmosphere models globally.

Contents

1 The FMS Driver and Sea Ice Coupling

FMS is a general framework for coupling global geophysical models and running them on multi-processing platforms. Some relevant aspects of FMS will be presented here. FMS allows models with different latitude/longitude grids to communicate conservatively through an exchange grid formed from the union of the bounding latitude and longitude lines of the communicating models. Thus, the atmosphere, ocean, and sea ice models may all be cast on separate grids. The FMS driver program is responsible for translating needed information from one grid to another through their mutual exchange grid so that each model sees only its own grid. (However, the ocean and ice models must agree upon the global land distribution.) Fluxes between the atmosphere and sea ice and between the atmosphere and land are partly calculated on the exchange grids. The sea ice model calculates ocean/sea ice fluxes on its own grid and they are subsequently translated by the driver to the ocean grid.

The FMS driver calls five ice model subroutines with arguements on the ice model grid (see Fig. 1). These five subroutines and their arguments define the standard ice model interface. Subroutine **ice_model_init** is called to read the the ice model restart (INPUT/ice_model.res), if it exists, initialize data structures and diagnostics, and set up the multi-processing domains. The driver contains two loops which define the coupling frequency of the ice with the atmosphere (the "fast" inner loop) and the ocean (the "slow" outer loop). At the top of the slow loop, subroutine **ice_bottom_to_ice_top** is called to set ice/ocean surface properties: temperature, albedo, roughness, etc. In the fast loop subroutine **update_ice_model_fast** is called to compute ice temperatures and surface fluxes. The fast timescale is determined by vertical mixing in the atmosphere and **update_ice_model_fast** is actually called between the downward and upward passes of the atmosphere model's tridiagonal solver. There is some complexity in this fast loop involved with the implicit timestepping of both the surface air and skin temperatures. The protocol for timestepping these temperatures is in fact a generalization of the tridiagonal technique. At the end of the fast loop, subroutine **update_ice_model_slow** is called to average the fluxes, compute sea ice mass and temperature changes, compute sea ice motion, transport ice, and compute fluxes at the ice-ocean interface. Finally, after both loops have been exited, subroutine **ice_model_end** is called to write the restart file (RESTART/ice_model.res). Fluxes and properties passed between the ice and other model components by the FMS driver are described in appendix A.

The remaining sections of this document describe the sea ice model functionally. The next section presents the processes that determine the temperature and mass changes of ice at a point in a particular thickness category independently of ice in other locations or thickness categories. The following section describes the determination of ice motion. The final section discusses transport of ice and ice properties and the redistribution of ice between thickness categories.

2 Thermodynamics

The sea ice vertical thermodynamics has a three layer structure similar to that of the Semtner model (Fig. 2). An optional snow layer overlies two ice layers of equal thicknesses. The snow has no heat capacity, the upper ice layer has sensible and latent heat capacity (i.e. brine pockets), and the lower ice layer has only sensible heat capacity. Thus there are four temperatures: one for each of the two ice layers, a diagnostically calculated surface temperature which is contrained to be at or below the sea ice freezing temperature, and the temperature of the ice/ocean interface held fixed at the freezing temperature of seawater (based upon the ocean mixed layer salinity).

When there is no snow cover, a fraction of the unreflected shortwave radiation penetrates the ice and is absorbed in the upper ice layer or the ocean mixed layer below the ice. The lower ice layer does not absorb solar radiation. The model employs a simple albedo scheme following Bettge et al (1996). Four albedos are used for snow/ice surfaces which are dry/melting. Dry snow and ice albedos are specified and a fixed decrement for melting conditions is applied.

Sea ice is formed by three mechanisms. The first is *frazil* formed in the ocean under the constraint that the ocean water temperature be above freezing. The ocean passes frazil as a (negative) energy; the ice model converts this into appropriate water and salt fluxes. *Congelation ice* is formed at the ice/ocean interface when its heat balance is negative. And *snow ice* is formed through an adjustment that converts snow below the waterline to sea ice. Melting can occur at either the upper or lower interface to balance their respective heat budgets while maintaining their respective temperature constraints.

The following two sections give details of the ice temperature and mass change calculations.

2.1 Temperature calculation

As mentioned above, a complicating factor in the atmosphere/sea ice heat flux calculation is that the surface air temperature and surface temperature are both timestepped implicitly, so that the air/ice flux is a function of the two temperatures at the current timestep. The scheme for this is described by Held (1998). First, the ice-atmosphere flux is linearized as follows:

$$F_s(T_a, T_s) = F_s(\hat{T}_a, \hat{T}_s) + \frac{\partial F_s}{\partial T_a}(T_a - \hat{T}_a) + \frac{\partial F_s}{\partial T_s}(T_s - \hat{T}_s) \tag{1}$$

Where $\frac{\partial F_s}{\partial T_a}$ and $\frac{\partial F_s}{\partial T_s}$ are evaluated with the temperatures at the previous timestep (denoted by hats). Ignoring terms not involving the surface flux, the prognostic equation for surface air temperature is

$$C_a(T_a - \hat{T}_a)/\Delta t = F_s \tag{2}$$

This is solved for $T_a - \hat{T}_a$ which is subsequently used in eqn. 1 to obtain the surface flux as a function of the new surface temperature, T_s,

$$F_s = A + BT_s \tag{3}$$

where

$$A = (F_s(\hat{T}_a, \hat{T}_s) - \hat{T}_s \frac{\partial F_s}{\partial T_s}) / \left(1 - \frac{\partial F_s}{\partial T_a}\frac{\Delta t}{C_a}\right) \tag{4}$$

and

$$B = \frac{\partial F_s}{\partial T_s} / \left(1 - \frac{\partial F_s}{\partial T_a}\frac{\Delta t}{C_a}\right) \tag{5}$$

The A/B pairs for each flux treated implicitly are passed to the ice model. The ice model then computes the new surface temperature and completes the flux computation using eqn. 3.

The conductive flux to the surface is

$$K_{1/2}(T_1 - T_s) \tag{6}$$

where

$$K_{1/2} \equiv \frac{4K_iK_s}{K_sh_i + 4K_ih_s} \tag{7}$$

is the effective conductive coupling of the snow/ice layer between the surface and the upper layer ice temperature $h_i/4$ beneath the snow/ice interface. See Table 1 for other symbol definitions. $K_{1/2}$ is determined by balancing the conductive flux of heat through the ice to the snow/ice interface with the conductive flux of heat through the snow away from the inteface. Notice, that it also applies when $h_s = 0$. As in the Semtner model, the ice conductivity is assumed to be constant rather than a function of temperature and salinity as it is in more sophisticated models (Maykut and Untersteiner, 1971; Bitz and Lipscomb, 1998). Now equating the fluxes to and from the ice upper surface, (3) and (6), gives the surface temperature

$$T_s = \frac{K_{1/2}T_1 - A}{K_{1/2} + B}. \tag{8}$$

Because the upper layer of ice contains liquid water (brine), the ice temperature calculation is somewhat more complicated than linear diffusion. The energy of the upper ice can be written

$$E(T, S) \equiv C(T + \mu S) - L(1 + \mu S/T) \tag{9}$$

where C, L, and T are the heat capacity, latent heat of fusion, and temperature of the ice, respectively, and μ is a constant equal to minus the freezing temperature (in ^oC) divided by the salinity. S is the bulk salinity of the ice (the salinity of the brine obtained by melting all of the ice). All the the salt is considered to be contained in liquid brine pockts. So a fraction, $-\mu S/T$, of the ice is actually liquid in order that the brine, with a salinity of $S/(-\mu S/T) = T/\mu$, can be in equilibrium with the solid ice at temperature T. Put another way: The ambient temperature, T, is always the freezing temperature of the brine. Differentiating (9) leads to a prognostic equation for upper layer temperature having the variable heat capacity form:

$$\frac{\rho_i h_i}{2}(C + \frac{L\mu S}{T_1^2})\frac{dT_1}{dt} = K_{1/2}(T_s - T_1) + K_{3/2}(T_2 - T_1) + I \tag{10}$$

where I is the surface penetrating solar radiation absorbed by the ice and

$$K_{3/2} \equiv 2K_i/h_i \tag{11}$$

is the conductive coupling between the two ice temperature points. The prognostic ice temperature equation for the fixed heat capacity ice lower layer is

$$\frac{\rho_i h_i}{2}C\frac{dT_2}{dt} = K_{3/2}(T_1 - T_2) + 2K_{3/2}(T_f - T_2) \tag{12}$$

where T_f is the temperature of the ice/ocean interface which is fixed at the (salinity dependent) seawater freezing temperature.

The equations (8), (10), and (12) are discretized and solved using the technique of Winton (2000). If the surface temperature is greater than the freezing temperature of snow (when there is snow cover) or sea ice (when there is none), the surface temperature is fixed at the melting temperature of snow or sea ice, respectively, rather than using (8). An energy flux

$$M_s = K_{1/2}(T_1 - T_s) - (A + BT_s) \tag{13}$$

is then applied toward surface melting (thereby balancing the surface energy budget). Likewise, the energy for bottom melting (or freezing, if negative)

$$M_b = F_b - 4K_i(T_f - T_2)/h_i \tag{14}$$

serves to balance the difference between the oceanic heat flux to the ice bottom, F_b, and the conductive flux of heat upward from the bottom. F_b is a linear function of the ice-ocean temperature difference

$$F_b = K_o(T_o - T_f) \tag{15}$$

where T_o is the sea surface temperature.

2.2 Ice and snow mass changes

In addition to calculating ice temperature changes, the ice model must also readjust the sizes of the snow and ice layers (1) to accomodate mass fluxes at the upper and lower surfaces, (2) to convert snow below the waterline to ice, and (3) to equalize the thickness of the two ice layers. It is convenient to apply mass increasing changes first. Snow is added to the snow layer and ice from two sources is added to the bottom layer. The first source is freezing at the ice ocean interface from (23) when M_b is negative. In this case, the bottom layer thickness, h_2 (initially, $h_2 = h_1 = h_i/2$) is increased by

$$\Delta h_2 = M_b \Delta t / E_2(T_f, S) \tag{16}$$

where

$$E_2(T, S) \equiv C(T + \mu S) - L \tag{17}$$

is the enthalpy of the lower, fixed heat capacity, ice. Notice that, even though it does not contain brine, the lower ice has the melting temperature of saline ice. Frazil from the ocean mixed layer, possibly formed to balance heat fluxes through leads when the mixed layer temperature falls to seawater freezing temperature, is added to the bottom of the ice along with the portion frozen at the ice/ocean interface. Both are added at seawater freezing temperature, changing the lower layer temperature to

$$T_{2new} = (\Delta h_2 T_f + h_2 T_2)/(\Delta h_2 + h_2) \tag{18}$$

Following, positive mass changes to the snow and ice layers, surface and bottom melting is applied. The surface melting that occurs to balance the (upper) surface energy budget described above reduces the snow thickness by

$$\Delta h_s = -min\left\{M_s \Delta t / L, h_s\right\} \tag{19}$$

the upper ice thickness by

$$\Delta h_1 = -min\left\{max\left[\frac{M_s \Delta t - L h_s}{-E(T_1, S)}, 0\right], h_1\right\} \tag{20}$$

and the lower ice thickness by

$$\Delta h_2 = -min\left\{max\left[\frac{M_s \Delta t - L h_s + E(T_1, S) h_1}{-E_2(T_2, S)}, 0\right], h_2\right\} \tag{21}$$

An excess melt energy of energy

$$max\left[M_s\Delta t - Lh_s + E(T_1,S)h_1 + E_2(T_2,S)h_2, 0\right] \tag{22}$$

is applied to the ocean mixed layer. Similarly, if M_b is positive (bottom melting), the bottom ice thickness is reduced by

$$\Delta h_2 = -min\left\{\frac{M_b\Delta t}{-E_2(T_2,S)}, h_2\right\} \tag{23}$$

the upper ice thickness by

$$\Delta h_1 = -min\left\{max\left[\frac{M_b\Delta t + E_2(T_2,S)h_2}{-E(T_1,S)}, 0\right], h_1\right\} \tag{24}$$

and the snow thickness by

$$\Delta h_s = -min\left\{max\left[\frac{M_b\Delta t + E_2(T_2,S)h_2 + E(T_1,S)h_1}{L}, 0\right], h_s\right\} \tag{25}$$

with excess melt energy of

$$max\left[M_b\Delta t - Lh_s + E(T_1,S)h_1 + E_2(T_2,S)h_2, 0\right] \tag{26}$$

applied to the ocean mixed layer.

After the above adjustments have been made to the thickness of the snow and ice layers, two internal adjustments are made. The first converts snow below the waterline

$$\Delta h_s = -max\left[\left(h_s - \frac{\rho_w - \rho_i}{\rho_s}h_i\right)\frac{\rho_i}{\rho_w}, 0\right] \tag{27}$$

into upper layer ice

$$\Delta h_1 = max\left[\left(h_s - \frac{\rho_w - \rho_i}{\rho_s}h_i\right)\frac{\rho_s}{\rho_w}, 0\right] \tag{28}$$

where ρ_w is the density of seawater. Now the temperature of the upper ice layer must be adjusted to account for the incorporation of zero-heat-capacity snow. This is done based upon enthalpy conservation, that is, by determining the temperature of the new upper layer that gives an enthalpy equal to the average enthalpies of the old upper layer and the added snow. For this purpose we note that the snow enthalpy, $-L = E_2(-\mu S, S)$. Hence an expression for new upper ice temperature when adding lower layer ice can also be used for adding snow simply by replacing T_2 with $-\mu S$. We now derive this expression which will also be used below for evening the ice layers.

Using (9) and (17) we can write the enthalpy of a new upper layer formed from a fraction f_1 of upper ice and $1 - f_1$ of lower ice

$$E(T_{1new}) = f_1 E(T_1) + (1 - f_1)E_2(T_2) \tag{29}$$

Solving for this for T_{1new} gives

$$T_{1new} = \frac{\bar{T} - \sqrt{\bar{T}^2 + 4\mu SL/C}}{2} \tag{30}$$

where

$$\bar{T} \equiv f_1 \left(T_1 - \frac{L}{C} \frac{\mu S}{T_1} \right) + (1 - f_1) T_2 \tag{31}$$

The final adjustment evens the upper and lower layers ice layers to maintain the layer structure depicted in Fig. 2. When shifting mass between the two layers, the temperature of the donating layer remains unchanged, while the temperature of the receiving layer is determined from averaging the enthalpies of the two ice portions. Equation (30) is used for determining the new temperature of an upper layer when lower layer ice is added to it. Averaging enthalpies for a new lower layer formed from a fraction f_1 of upper layer ice and $1 - f_1$ of lower layer ice gives a new lower layer temperature

$$T_{2new} = \bar{T} \tag{32}$$

where $\bar{T}$ is given by (31).

When upper layer ice is converted to lower layer ice it is possible for the new lower layer temperature to exceed the ice melting temperature, $-\mu S$. To avoid this condition, this excess sensible energy is used to melt equal thicknesses from the upper and lower layers.

Sea ice may be given a fixed non-zero bulk salinity which is used for calculating ice/ocean salt fluxes. This salinity is distinct from the salinity used for the brine calculation. The brine calculation salinity represents the salinity that is important for brine storage, that is, the salinity of the upper optical depth of the ice at the end of the melting season. The bulk salinity represents the salinity of melting and freezing ice. When sea ice freezes and melts, salt is taken from or given to the ocean to maintain the specified ice salinity. When snow below the waterline is converted to sea ice, salt is taken from the ocean to bring its salinity up to the specified value.

3 Dynamics

Ice motion is calculated from the equation

$$m \left\{ \frac{\partial \mathbf{v}}{\partial t} + f\mathbf{k} \times \mathbf{v} + g\nabla\eta \right\} = \nabla \cdot \boldsymbol{\sigma} + \Sigma_k c_k \left(\boldsymbol{\tau}_a + \boldsymbol{\tau}_w \right) \tag{33}$$

where m is the mass of ice and snow per m^2 of ocean surface, η is the sea level, $\boldsymbol{\sigma}$ is the ice internal stress tensor, and $\boldsymbol{\tau}_a$ and $\boldsymbol{\tau}_w$ are the surface stresses of the atmosphere and ocean on the ice, respectively. Notice that these surface stresses are scaled by the concentration of ice cover ($\Sigma_k c_k$ is the sum of the concentrations of the ice categories). The internal stress term, likewise, scales with the ice thickness through the ice strength given below.

The ice-ocean stress is calculated by the ice model according to the drag law

$$\boldsymbol{\tau}_w = c_w \rho_w \left| \mathbf{v}_w - \mathbf{v} \right| \left\{ (\mathbf{v}_w - \mathbf{v}) \, cos\theta + \mathbf{k} \times (\mathbf{v}_w - \mathbf{v}) \, sin\theta \right\} \tag{34}$$

where c_w is the water drag coefficient, ρ_w is the density of seawater, $\mathbf{v}_w$ is the current at the ocean's topmost grid cell, and θ is a turning angle to account for unresolved Ekman shear in the current between the top cell depth and the surface.

The stress tensor divergence is given by the generalized orthogonal coordinate form

$$\begin{aligned} \nabla \cdot \boldsymbol{\sigma} = \frac{1}{h_1 h_2} \quad & \left\{ \left[\tfrac{\partial (h_2 \sigma_{11})}{\partial \xi_1} + \tfrac{\partial (h_1 \sigma_{12})}{\partial \xi_2} + \sigma_{12} \tfrac{\partial h_1}{\partial \xi_2} - \sigma_{22} \tfrac{\partial h_2}{\partial \xi_1} \right] \mathbf{e}_1 \right. \\ & \left. + \left[\tfrac{\partial (h_2 \sigma_{12})}{\partial \xi_1} + \tfrac{\partial (h_1 \sigma_{22})}{\partial \xi_2} + \sigma_{12} \tfrac{\partial h_2}{\partial \xi_1} - \sigma_{11} \tfrac{\partial h_1}{\partial \xi_2} \right] \mathbf{e}_2 \right\} \end{aligned} \tag{35}$$

where ξ_i are the nondimensional grid variables, h_i are the metric factors ($h_i = \partial x_i / \partial \xi_i$), and $\mathbf{e}_i$ are unit vectors in the two horizontal directions. This expression is simplified from a similar three-dimensional expression given by Murray and Reason (2000).

The viscous-plastic rheology is defined by the tensor stress-rate of strain relationship (Hibler, 1979)

$$\sigma_{ij} = 2\eta\epsilon_{ij} + (\zeta - \eta)\epsilon_{kk}\delta_{ij} - P\delta_{ij}/2 \tag{36}$$

where ϵ_{ij} is the strain rate tensor, P is the ice strength, and η and ζ are the bulk and shear viscositites, respectively.

The rate of strain tensor is given by the generalized orthogonal coordinate form

$$\epsilon_{11} = \frac{1}{h_1}\frac{\partial u_1}{\partial \xi_1} + \frac{u_2}{h_1 h_2}\frac{\partial h_1}{\partial \xi_2} \tag{37}$$

$$\epsilon_{12} = \frac{h_2}{2h_1}\frac{\partial}{\partial \xi_1}\left(\frac{u_2}{h_2}\right) + \frac{h_1}{2h_2}\frac{\partial}{\partial \xi_2}\left(\frac{u_1}{h_1}\right) \tag{38}$$

$$\epsilon_{22} = \frac{1}{h_2}\frac{\partial u_2}{\partial \xi_2} + \frac{u_1}{h_1 h_2}\frac{\partial h_2}{\partial \xi_1} \tag{39}$$

where u_i are the velocity components in the two horzontal directions. This expression is also a special case of a more general form given by Murray and Reason (2000).

The ice viscosities are

$$\zeta = \frac{P}{2\Delta} \quad \eta = \zeta/e^2 \tag{40}$$

where

$$\Delta = \left\{\left(\epsilon_{11}^2 + \epsilon_{22}^2\right)\left(1 + e^2\right) + 4e^{-2}\epsilon_{12}^2 + 2\epsilon_{11}\epsilon_{22}\left(1 - e^{-2}\right)\right\}^{1/2} \tag{41}$$

and e is the plastic yield curve eccentricity.

The ice strength is given by the standard parameterization (Hibler, 1979)

$$P = P^* \Sigma_k (c_k h_k) exp\left[-c^*\left(1 - \Sigma_k c_k\right)\right] \tag{42}$$

where P^* and c^* are constants and the summations Σ_k are taken over the ice thickness categories.

Hunke and Dukowicz show that the solution to (36) may be determined as the steady state of

$$\frac{1}{E}\frac{\partial \sigma_{ij}}{\partial t} + \frac{1}{2\eta}\sigma_{ij} + \frac{\eta - \zeta}{4\eta\zeta}\sigma_{kk}\delta_{ij} + \frac{P}{4\zeta}\delta_{ij} = \epsilon_{ij} \tag{43}$$

This equation is subcycled in the model along with (33) until the elastic waves have damped out and the viscous-plastic rheology is recovered. Hunke and Dukowicz discuss the setting of E and the timestepping procedure in detail. Three alterations to the Hunke and Dukowicz scheme are made in the current model: (1) standard B-grid differencing techniques are used – reducing the computational cost by roughly a factor of four, (2) metric terms are retained in the stress divergence and strain rate tensor, and (3) the viscosities, η and ζ, are recalculated at every subcycling timestep.

4 Transport and Thickness Category Redistribution

The timestepping of ice thermodynamic quantities is divided into four substeps: (1) Heat fluxes and temperature changes are calculated. (2) The (upper and lower) surface heat fluxes from the first step are used along with precipitation and evaporation fluxes to calculate mass fluxes which are then applied; temperatures are updated in an enthalpy conserving manner. (3) Grid extensive quantities [c_k, $c_k h_{sk}$, $c_k h_k$, $c_k h_k (T_{k1} - (\mu S_i L_i / C_i)/T_{k1})$, and $c_k h_k T_{k2}$] are formed from the fields to be advected [c_k, h_{sk}, h_k, T_{k1}, and T_{k2}], these quantities are advected using an upstream evaluation of the generalized horizontal coordinate transport term,

$$\frac{1}{h_1 h_2} \left\{ \frac{\partial}{\partial \xi_1} (h_2 u_1 E) + \frac{\partial}{\partial \xi_2} (h_1 u_2 E) \right\}. \tag{44}$$

(4) The thickness distribution is adjusted as described below, and finally, the orignal fields are reconstructed from the above grid extensive quantities.

The ice model maintains n partitions, the first for open water and the second through nth for ice thickness categories in order of increasing thickness. Each ice category has a minimum thickness – the minimum for the first thickness category is zero. For each category, aside from the thickest, the minimum thickness of the next thicker category serves as a maximum thickness. The thickest category has no maximum thickness. Frazil (ice formed in the ocean and passed to the ice model) is added to the thinnest ice thickness category that, when combined with the open water category has a non-zero concentration. The concentration of this category is then expanded to include any of the formerly open water, so that no open water remains after frazil is formed.

The redistribution of ice betweeen thickness categories follows a simple algorithm from Igor Polyakov [1] (personal communication). When the total ice concentration at a grid point is greater than unity following transport, a pass through the thickness categories from thin to thick reduces the total ice concentration back to one as follows. The excess concentration is initally subtracted from the thinnest category. If the concentration of this category becomes negative, its ice properties are combined conservatively with the next thicker category and its (negative) concentration added to the concentration of the thicker category. This procedure continues until the concentration of the receiving category is not negative. Following this, a pass is made through the categories from thin to thick moving ice which has exceeded its upper thickness limit to the next thicker category. Finally, a pass is made from thick to thin categories moving ice which has fallen below its category minimum thickness to the next thinner category.

An example is useful. Consider the situation in a grid cell with convergence and melting depicted at the top of Fig. 3. Because of the convergence the sum of the concentrations is greater than one. The first adjustment eliminates the open water and reshapes the category 1 ice to reset the total concentration to one. The pass from thin to thick reveals that the first ice category is now thicker than the category 1 maximum (which equals the category 2 minimum) so the category 1 ice is merged into category 2. The thick to thin pass shows the category 3 ice has melted below its minimum and so it also is merged with the category 2 ice. The final ice distribution, a single category 2 block, satisfies both areal and thickness constraints.

[1] Igor Polyakov made important contributions to SIS including this scheme.

5 References

- Bettge, T.W., J.W. Weatherly, W.M. Washington, D. Pollard, B.P. Briegleb, and W.G. Strand Jr., 1996: *The NCAR CSM Sea Ice Model.* NCAR Technical Note, TN-425+STR, 25 pp.

- Bitz, C.M., and W. H. Lipscomb, 1999: A new energy-conserving sea ice model for climate study. *J. Geophys. Res.*, **104**, 15669–15677.

- Held, I., 1998: Surface fluxes, implicit vertical diffusion, and the exchange grid. GFDL internal note.

- Hibler, W.D. III, 1979: A dynamic thermodynamic sea ice model. *J. Phys. Oceangr.*, **9**, 817–846.

- Hunke, E.C., and J.K. Dukowicz, 1997: An elastic-viscous-plastic model for sea ice dynamics. *J. Phys. Oceangr.*, **27**, 1849–1867.

- Maykut, G.A., and N. Untersteiner, 1971: Some results from a time-dependent thermodynamic model of sea ice. *J. Geophys. Res.*, **76**, 1550–1575.

- Murray, R.J., and C.J.C. Reason, 2000: A curvilinear version of the Bryan-Cox ocean model. Submitted to *J. Comp. Phys.*.

- Ono, N., 1967: Specific heat and heat of fusion of sea ice, in *Physics of Snow and Ice*, H. Oura, ed., v. I, 599–610.

- Semtner, A.J., 1976: A model for the thermodynamic growth of sea ice in numerical investigations of climate. *J. Phys. Oceangr.*, **6**, 27–37.

- R.D. Smith, S. Kortas, and B. Meltz, 1995: Curvilinear coordinates for global ocean models. Technical report LA-UR-95-1146, Los Alamos National Laboratory.

- Winton, M., 2000: A reformulated three-layer sea ice model. *J. Atmos. Ocean. Tech.*, **17**, 525–531.